atively sloped, or positively sloped. Thus the price-consumption curve in Figure 2.4.1 reflects commodity demand that is first (at higher prices) elastic, becomes unitary at a point, and is inelastic thereafter.

2.5 CONCLUSION

The basic principles of consumer behavior and of demand have now been developed. In the following two chapters various important, but subsidiary, topics are analyzed using the tools introduced in Chapters 1 and 2. The fundamental conclusion of this chapter is explained more fully and one special exception is noted, but this conclusion remains as fundamental as ever: if consumers behave so as to maximize satisfaction from a limited money income, quantity demanded will vary inversely with price.

SUGGESTED READINGS

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2. HENDERSON, JAMES M. AND QUANDT, RICHARD E. *Microeconomic Theory: A Mathematical Approach,* pp. 12–24. New York: McGraw-Hill Book Co., Inc., 1958. [Elementary math necessary.]
3. SAMUELSON, PAUL A. *Foundations of Economic Analysis,* pp. 96–100. Cambridge: Harvard University Press, 1947. [Advanced math necessary.]

Chapter 3 TOPICS IN CONSUMER DEMAND

3.1 INTRODUCTION

The theory of consumer behavior was developed in Chapter 2, and it was shown that an individual consumer demand curve normally slopes downward to the right—that quantity demanded varies inversely with price. This chapter presents a closer analysis of consumer demand and of market demand for related commodities.

3.2 SUBSTITUTION AND INCOME EFFECTS

A change in the nominal price of a commodity actually exerts two influences on quantity demanded. In the first place, there is a change in *relative* price—a change in the terms at which a consumer *can* exchange one good for another. The change in relative price alone leads to a substitution effect. Second, a change in the nominal price of a good (nominal income remaining constant) causes a change in *real* income, or in the size of the bundle of goods and services a consumer can buy. If the nominal price of one good falls, all other nominal prices remaining constant, the consumer's real income rises because he can now buy more, either of the good whose price declined or of other goods. In other words, his level of satisfaction must increase. The change in the level of real income may or may not—depending upon the consumer's preference map—cause a significant change in his pattern of consumption. In any event, the change in real income leads to an income effect upon quantity demanded.

3.2.a—The Substitution Effect in the Case of a Normal Good

When the price of one good changes, the prices of other goods and money income remaining constant, the consumer moves from one equilibrium point to another. In normal circumstances, if the price of a good diminishes, more of it is bought; if its price increases, fewer units are taken. The overall change in quantity demanded from one equilibrium position to another is referred to as the *total effect.*

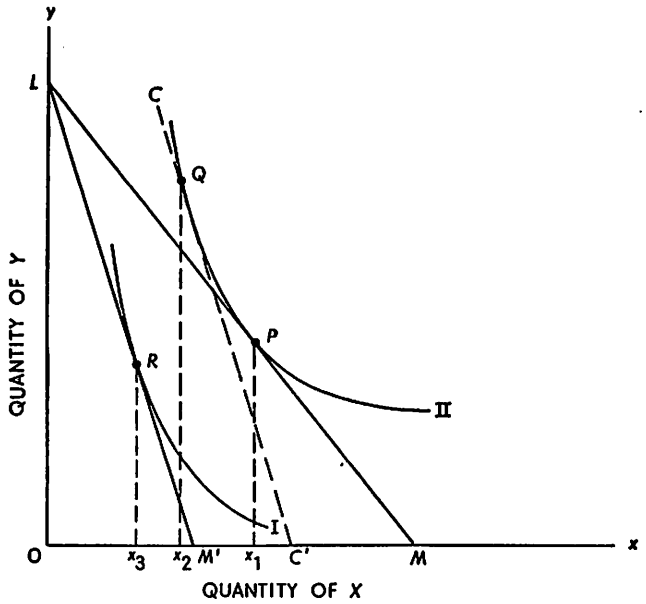

FIGURE 3.2.1

SUBSTITUTION AND INCOME EFFECTS FOR A NORMAL GOOD IN CASE OF A PRICE RISE

Definition: the total effect of a price change is the total change in quantity demanded as the consumer moves from one equilibrium to another.

The total effect of a price change is illustrated in Figure 3.2.1. The original price ratio is indicated by the slope of the budget line *LM*. The consumer attains equilibrium at point *P* on indifference curve *II*, purchasing Ox_1 units of *X*. When the price of *X* rises, as indicated by shifting the budget line from *LM* to *LM'*, the consumer moves to a new equilibrium position at *R* on indifference curve *I*. At this point he purchases Ox_3 units of *X*. The total effect of the price change is indicated by the movement from *P* to *R*, or by the reduction in quantity demanded from Ox_1 to Ox_3. In other words, the total effect is $Ox_1 - Ox_3 = x_1 x_3$. This is called a negative total effect because quantity demanded is reduced by $x_1 x_3$ units.

The total effect of a price change, however, can be decomposed into two effects, the *substitution effect* and the *income effect.* Let us first examine the substitution effect. Consider Figure 3.2.1. When the price of *X* increases, the consumer suffers a decline in real income, as indicated by the movement from indifference curve *II* to indifference curve *I*. Suppose that coincident with the price rise the consumer were given an amount of (additional) money income just sufficient to compensate him for the loss in real income he would otherwise sustain.

That is, he is given a compensatory payment just sufficient to enable him to remain on indifference curve *II* under the *new* price regime.

Graphically, this compensation is shown by constructing a fictitious budget line tangent to the *original* indifference curve, but whose slope corresponds to the *new* price ratio. The dashed line *CC'* in Figure 3.2.1. is the fictitious budget line for this example—it is tangent to the original indifference curve *II* at point *Q;* but it is parallel to the new budget line *LM'*, thereby reflecting the new price ratio.

The substitution effect is represented by the movement from the original equilibrium position at *P* to the imaginary equilibrium position at *Q*, both points being situated on the original indifference curve. In terms of quantity, the substitution effect is the reduction in quantity demanded from Ox_1 to Ox_2, or by $x_1 x_2$ units.

Definition: the substitution effect is the change in quantity demanded resulting from a change in relative price after compensating the consumer for his change in real income. In other words, the substitution effect is the change in quantity demanded resulting from a change in price when the change is restricted to a movement along the original indifference curve, thus holding real income constant.

The substitution effect in the case of a price decline is illustrated in Figure 3.2.2. The original equilibrium is point *P* on indifference curve *I*,

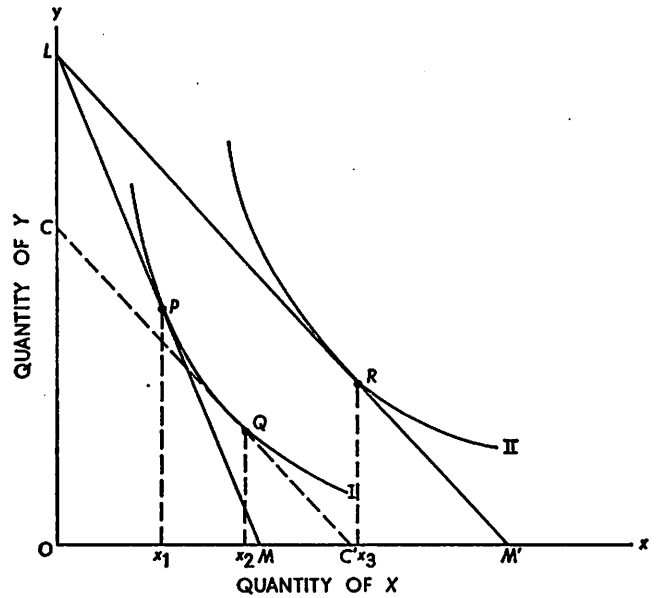

FIGURE 3.2.2

SUBSTITUTION AND INCOME EFFECTS FOR A NORMAL GOOD IN CASE OF A PRICE DECREASE

the price ratio being indicated by the original budget line LM. The price of X now declines to that indicated by the slope of LM'. In the absence of a compensatory payment, the consumer would enjoy an increase in real income, moving to equilibrium on indifference curve II. In this case, we compensate by imagining a decrease in money income of an amount just sufficient to maintain real income constant at the new price ratio. Graphically, this is illustrated by the dashed line CC'.

As a result of the price change alone, real income held constant, the consumer moves from the original equilibrium at P to the imaginary equilibrium at Q. The movement from P to Q along the original indifference curve represents the substitution effect. In quantity units, it is the expansion of quantity demanded from Ox_1 to Ox_2.

Comparing the cases in Figures 3.2.1. and 3.2.2., one readily sees that the substitution effect is *always negative*—an increase in the price of X, real income constant, leads to a substitution of Y for X, while a fall in the price of X under the same circumstances induces a substitution of X for Y. Expressed somewhat differently, quantity demanded always varies inversely with price for movements along an indifference curve.

3.2.b—The Income Effect in the Case of a Normal Good

In determining the substitution effect, one is constrained to movements along the original indifference curve. However, the total effect of a price change, money income and the prices of other commodities held constant, always entails a shift from one indifference curve to another, or a change in real income.

Definition: the income effect is the change in quantity demanded resulting exclusively from a change in real income, all prices and money income held constant.

Consider Figure 3.2.1. When the price of X rises, as indicated by the shift of the budget line from LM to LM', the consumer attains his new equilibrium on indifference curve I. The movement from P to Q along indifference curve II represents the substitution effect. Now let the consumer's real income fall from the level represented by the fictitious budget line CC'. The movement from the imaginary equilibrium position Q on indifference curve II to the actual new equilibrium position R on indifference curve I indicates the income effect. Since CC' and LM' are parallel, the movement does not involve a change in relative prices. It is a real-income phenomenon.

Real income declines as a result of the rise in the price of X. The

reduction in quantity demanded from Ox_2 to Ox_3 measures the change in purchases attributable exclusively to the decline in real income, the change in relative price already having been accounted for by the substitution effect.

Similarly, in Figure 3.2.2, the decline in the price of X leads to an increase in real income. The substitution effect accounts for the movement from P to Q, and the income effect is represented by the movement from Q to R. Real income increases as a result of the price decrease, and quantity demanded increases from Ox_2 to Ox_3, *exclusively* as a result of the increase in real income.

From either graph one may readily see that the total effect of a price change is the sum of the substitution and income effects. In Figure 3.2.1, the total effect of the rise in the price of X is a reduction in quantity demanded from Ox_1 to Ox_3. The movement from Ox_1 to Ox_2 is attributable to the substitution effect and the movement from Ox_2 to Ox_3 is the income effect. The same reasoning applies, *mutatis mutandis*, for the total effect shown in Figure 3.2.2.

3.2.c—Normal Goods

As indicated by the subheadings above, our analysis has so far been restricted to the case of "normal" goods, but a "normal" good has not yet been defined. We now have the tools necessary for the definition.

Note from Figure 3.2.1 that when the price of a commodity rises, real income declines and the income effect causes a decrease in quantity demanded. On the other hand, a price decline (Figure 3.2.2) leads to an increase in real income and to an increase in quantity purchased attributable to the income effect. In both these cases, the income effect is *positive:* an increase in real income leads to an increase in quantity demanded and vice versa.

Definition: a normal good is one for which the income effect is positive.
Principle: a positive income effect reinforces the negative substitution effect. Thus for a normal good, quantity demanded always varies inversely with price. The law of demand applies to all normal goods.

The result of this section may be summarized as follows.[1]

Relationships: the total effect of a price change may be decomposed into a substitution effect and an income effect. The substitution effect is the change in quantity demanded attributable exclusively to a change in the price ratio. The substitution effect is always negative. The income effect is the change in quantity demanded attributable exclusively to a change in real income. For

[1] Mathematically, the substitution and income effects can be explained succinctly. Consider a consumer who may select the quantities $x_1, x_2, \ldots, x_n$ of n goods at fixed

normal goods, the income effect is positive. A positive income effect reinforces the negative substitution effect. Thus for normal goods, the demand curve always slopes downward to the right.

prices $p_1, p_2, \ldots, p_n$. Let his money income be

$$(3.1.1) \qquad M = \sum_{j=1}^{n} p_j x_j ,$$

and let

$$(3.1.2) \qquad u = u(x_1, x_2, \ldots, x_n)$$

be an ordinal index of preference for this consumer.

Maximizing (3.1.2) subject to the linear constraint (3.1.1) yields the n equations

$$(3.1.3) \qquad u_i = \lambda p_i, \qquad (i = 1, 2, \ldots, n)$$

where $u_i = \partial u / \partial x_i$ and λ is the Lagranean multiplier. Equations (3.1.3) and (3.1.1) together provide $n + 1$ equations to solve for the $n + 1$ unknowns $x_1, x_2, \ldots x_n$, and λ.

For the stability of equilibrium it is necessary and sufficient that the elements of

$$(3.1.4) \qquad [U] = \begin{bmatrix} 0 & u_i \\ u_j & u_{ij} \end{bmatrix} \qquad (i, j = 1, 2, \ldots, n)$$

be associated with a quadratic form that is negative definite under constraint. This, in turn, requires the successive bordered principal minors to alternate in sign.

The effect of a change in money income upon quantities demanded may be determined by taking the partial derivatives of (3.1.3) and (3.1.1) with respect to M (holding all prices constant) and solving the resulting system of equations by Cramer's rule. A typical solution term is

$$(3.1.5) \qquad \frac{\partial x_i}{\partial M} = \frac{\lambda U_i}{U} ,$$

where U_i is the cofactor of u_i in $[U]$ and U is the determinant of $[U]$.

The effect of a price change on quantities demanded is found by taking the partial derivatives with respect to (say) p_i and solving by Cramer's rule. A typical term is

$$(3.1.6) \qquad \frac{\partial x_i}{\partial p_i} = \frac{\lambda x_i U_i + \lambda U_{ii}}{U} ,$$

where U_{ii} is the cofactor of u_{ii} in $[U]$. Substituting (3.1.5) into (3.1.6), one obtains what is sometimes called the fundamental equation of value theory:

$$(3.1.7) \qquad \frac{\partial x_i}{\partial p_i} = -x_i \frac{\partial x_i}{\partial M} + \frac{\lambda U_{ii}}{U} .$$

(total effect) = (income effect) + (substitution effect).

For normal goods, $\partial x_i / \partial M > 0$. Further, since the successive bordered principal minors of $[U]$ must alternate in sign, $U_{ii}/U < 0$. Hence

$$(3.1.8) \qquad \frac{\partial x_i}{\partial p_i} = -x_i \frac{\partial x_i}{\partial M} + \frac{\lambda U_{ii}}{U} < 0$$

necessarily, for normal goods. The total effect, $\partial x_i / p_i$, can only be positive when $\dfrac{\partial x_i}{\partial M} < 0$ and

$$(3.1.9) \qquad \left| x_i \frac{\partial x_i}{\partial M} \right| > \left| \frac{\lambda U_{ii}}{U} \right| .$$

If condition (3.1.9) holds, one obtains the case of Giffen's paradox, discussed in Section

3.3. INFERIOR GOODS

"Normal" goods are given this name because in almost all cases the income effect is positive—this is the "normal" situation. In certain unusual cases, however, the income effect may cause a switch from margarine to butter, from dried to fresh vegetables. Thus an increase in real income may result in a decrease in the consumption of certain commodities. These commodities are called inferior goods.

Definition: an inferior good is one for which the income effect is negative.

3.3.a—Inferior Goods and Giffen's Paradox

An increase in real income may be attributable to an increase in money income, commodity prices remaining constant, or to a decline in prices, money income remaining constant. Figure 3.3.1 shows an increase in income from the level given by the budget line LM to that given by $L'M'$. The two budget lines are parallel, so no change in relative price has occurred. Real income increases from LM to $L'M'$ either by an increase in money income, prices constant, or by uniform percentage reduction in both prices.

In the change, the position of consumer equilibrium shifts from

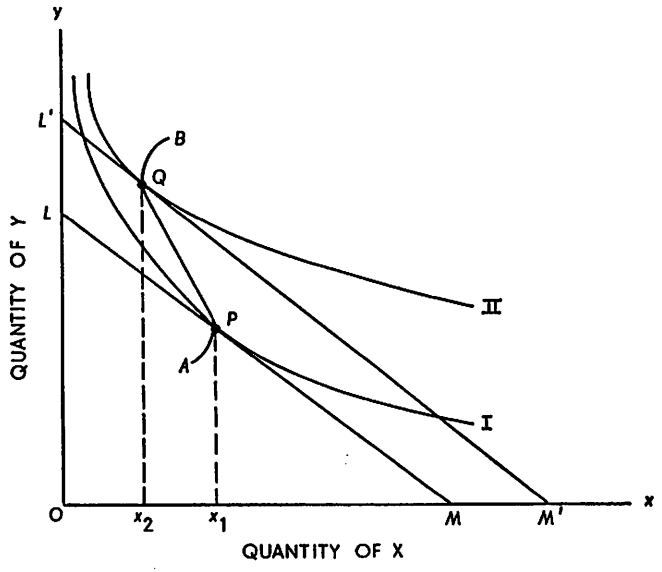

FIGURE 3.3.1

ILLUSTRATION OF AN INFERIOR GOOD

point P on indifference curve I to point Q on indifference curve II. As a result of the *increase* in real income at the constant *relative* prices, the quantity demanded of good X falls from Ox_1 to Ox_2. The income-consumption curve, over this range of real-income values, rises backward from P to Q; and the entire income-consumption curve might resemble the curve $APQB$.

Figure 3.3.1 illustrates an indifference map involving one inferior good (X). The income effect is negative, a rise in real income at a constant price ratio leading to a decline in quantity demanded. Similarly, if $L'M'$ is regarded as the original income level, LM represents a lower real income. In this case, a decline in real income would be accompanied by an increase in the quantity of X demanded.

Generally, the substitution effect of a price change is great enough to offset a negative income effect. But in one case, called Giffen's paradox, the income effect is so strong that it more than offsets the substitution effect. Thus a decline in price leads to a decline in quantity demanded and a rise in price induces a rise in quantity demanded. Figure 3.3.2 is an illustration of Giffen's paradox. The original price of X is given by the slope of LM. With given money income and a constant price of Y, the price of X falls to the level indicated by the slope of LM'. The position of consumer equilibrium shifts from point P on indifference curve I to point Q on indifference curve II. Over this range, the price-consumption curve is PQ, and throughout the entire range it

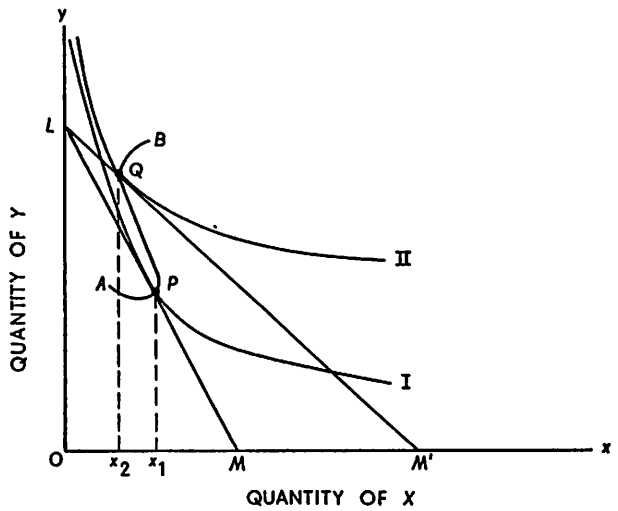

FIGURE 3.3.2
ILLUSTRATION OF GIFFEN'S PARADOX

might look like the curve $APQB$. In the case of Giffen's paradox, the price-consumption curve is *backward rising* over a certain range.

Definition: Giffen's paradox refers to a good whose quantity demanded varies directly with price. A good must be an inferior good to belong in this category; but not all inferior goods conform to the conditions of Giffen's paradox. The class of goods for which Giffen's paradox holds constitutes the only exception to the law of demand.

3.3.b—*Income and Substitution Effects for an Inferior Good*

The relationships described in subsection 3.3.a are shown more clearly by separating the total effect into its component parts. Figure 3.3.3 is an illustration of the income and substitution effects for an inferior good not subject to the conditions of Giffen's paradox.

In Figure 3.3.3, LM is the original budget line. The price of X falls so that the budget line is shifted to LM'. Equilibrium shifts from point P on indifference curve I to point R on indifference curve II. In the process, the quantity of X demanded *increases* from Ox_1 to Ox_3. This case, even though it involves an inferior good, does not violate the law of demand—the substitution effect more than counterbalances the negative income effect.

To see this, construct the fictitious budget line CC' showing the

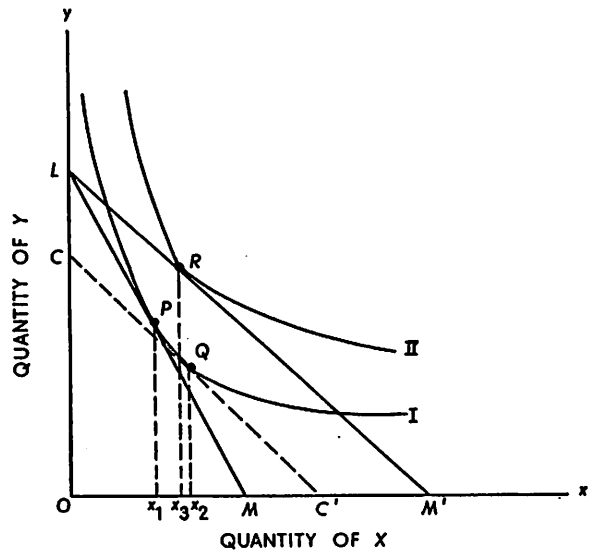

FIGURE 3.3.3
INCOME AND SUBSTITUTION EFFECTS FOR AN INFERIOR GOOD NOT SUBJECT TO GIFFEN'S PARADOX

old level of real income and the new price ratio. The movement from P to Q—or the increase in quantity demanded from Ox_1 to Ox_2—is the substitution effect. It is strongly negative, for the decrease in price results in a significant increase in quantity demanded, real income held constant. The income effect is also negative in this case, as is shown by the movement from Q to R or by the decrease in quantity demanded from Ox_2 to Ox_3.

Here we have a case in which the (always negative) substitution effect is partially offset by a negative income effect. But the negative income effect is not strong enough to cause quantity demanded to vary

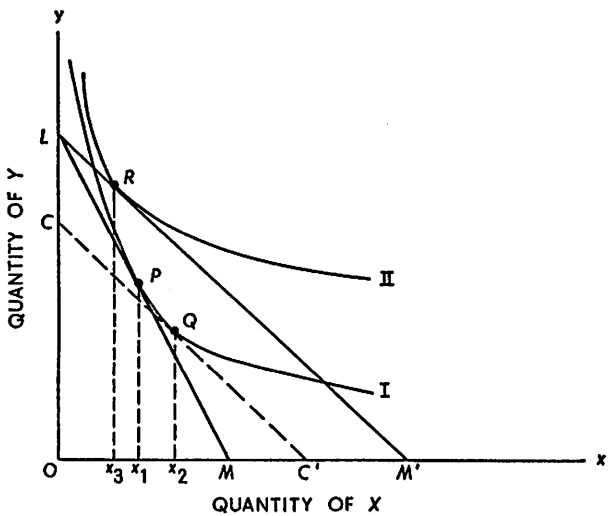

FIGURE 3.3.4

INCOME AND SUBSTITUTION EFFECTS FOR GIFFEN'S PARADOX WHEN PRICE DECLINES

directly with price. The law of demand holds, but demand is very inelastic over this range (as indicated by the steep upward slope of the price-consumption curve from P to R).

The law of demand fails to hold only in the case of Giffen's paradox, as illustrated in Figure 3.3.4. The original position of equilibrium is point P, where the budget line LM is tangent to indifference curve I. The price of X falls to that given by the slope of LM' and the ultimate new equilibrium is obtained at R on indifference II. First construct the fictitious budget line CC' showing the old level of real income and the new price ratio. The substitution effect is represented by the movement from P to Q along indifference curve I, or by the *increase* in quantity demanded from Ox_1 to Ox_2.

The income effect is represented by the movement from Q on I to R on II, or by the decrease in quantity demanded from Ox_2 to Ox_3. Adding the negative substitution and income effects together, the total effect of the price decline is the decrease in quantity demanded from Ox_1 to Ox_3. For a commodity such as this, the law of demand is not valid.[2]

3.3.c—*The Nature of Inferior Goods*

Whether a commodity is an inferior good or not has nothing to do with the total effect of a price change. It is strictly an income phenomenon. Giffen's paradox, on the other hand, relates only to certain inferior goods that violate the law of demand.

As an example, oleomargarine may be an inferior good, but it certainly does not belong to the Giffen's paradox category. As we have definitely seen in the past, a reduction in the price of oleomargarine leads to a substitution of margarine for butter. However, an increase in the real income of a family may cause that consumption unit to switch from margarine to butter. But in this case the income effect is not great enough to offset the substitution effect.

In all probability there are very few households in the United States or other advanced nations for which Giffen's paradox obtains. A negative income effect is not all that is required for this case—the good must also be very important in the entire family budget. The classic example is potatoes in nineteenth-century Ireland. The typical Irish peasant was so poor, it was said, that he spent almost all his cash income for the least expensive means of subsistence, potatoes.

Now suppose the price of potatoes falls. The same number of calories can now be bought for less expenditure on potatoes, so some money is available for green vegetables and perhaps meat. But these items also contain calories, so the consumption of potatoes can actually be reduced. Thus Giffen's paradox is obtained—a reduction in price leads to a reduction in quantity demanded.

Giffen's paradox is a bona fide exception to the law of demand. However, in the type of society with which we are presently concerned,

[2] In this text, and in almost all other usages, demand and the law of demand are defined in terms of constant nominal money income. Thus real income changes as one moves along a demand curve; Giffen's paradox can occur, and the law of demand is not universally valid. For certain uses, however, it is convenient to construct a demand curve based on constant real income (and, therefore, varying nominal money income). Such demand curves are called income-compensated demand curves. The "income effect" is, in effect, subtracted, leaving only the substitution effect. Such demand curves always slope downward to the right, irrespective of the type of good. For a thorough discussion of income-compensated demand curves, see Milton Friedman, "The Marshallian Demand Curve," *Journal of Political Economy*, LVII (1949), pp. 463–95.

Giffen's paradox is a rare phenomenon. It occurs in few consumer units and, within these units, for very few commodities. Thus when all individual demand curves are aggregated to obtain market demand curves, it is safe to assume that market quantity demanded varies inversely with price for every commodity.

3.4 SUBSTITUTION AND COMPLEMENTARITY

When an individual's demand schedule is constructed, his preference pattern, his nominal money income, and the nominal prices of related commodities are held constant. Thus a demand schedule shows the relationship between the nominal price of a commodity and the quantity of it demanded, all other demand influences held constant (or impounded in a *ceteris paribus* assumption). This partial equilibrium demand function is quite useful for some purposes, but much less useful for others. In some situations a general equilibrium view of the problem is required. So far as demand analysis is concerned, this means that one or more of the *ceteris paribus* assumptions must be relaxed.

More particularly, if the nominal prices of related commodities are allowed to vary, there will be definite repercussions on the quantity demanded of the commodity in question. By observing these repercussions, one is able to classify pairs of commodities as substitute or complementary goods. Historically, the first method of classification was based upon the *total effect* upon quantity demanded of good X resulting from a change in the price of good Y. After the appearance of Hicks' *Value and Capital* it was realized that a more accurate classification can be obtained by analyzing the *substitution effect* alone. But while the latter method is more accurate, it is also more difficult to utilize on an empirical level. Thus in actual problems the older and less precise method must usually be used. For that reason both classificatory schemes are presented in this section.

3.4.a—Classification by Cross Elasticities

If all prices are allowed to vary, the quantity of good X demanded depends not only upon its own price but upon the prices of related goods as well. Instead of a demand *curve* there is a demand *surface* such as those shown in Figures 3.4.1 and 3.4.2.

For illustrative purposes suppose good X is related to only one other commodity, good Y. Schematically, the demand function can no longer be written as $q = h(p)$. Instead, one must write $q_x = f(p_x, p_y)$,

where q and p represent quantity and price and the subscripts indicate the commodity in question.

The price elasticity of demand, or "own" elasticity, is

$$\eta_{xx} = \left(\frac{\Delta q_x}{q_x} \div \frac{\Delta p_x}{p_x}\right),$$

where Δ means "the change in." The direct price elasticity, in other words, is the proportional change in quantity demanded of good X resulting from a given proportional change in the price of good X. The elasticity formula is applicable whether the demand function has the form shown in the first or second equations. When the price of a related good enters the demand function, however, it is possible to define the price cross-elasticity of demand:

$$\eta_{xy} = \frac{\Delta q_x}{q_x} \div \frac{\Delta p_y}{p_y}.$$

The price cross-elasticity of demand is the proportional change in the quantity of X demanded resulting from a given proportional change in the price of the related good Y.

According to the cross-elasticity classification, goods X and Y are substitutes or complements according as the price cross-elasticity of demand is positive or negative. As trivial examples, consider the following. An increase in the price of pork, the price of beef remaining constant, will tend to augment the quantity of beef demanded; η_{xy} is positive and beef and pork are said to be substitute goods. On the other hand, an increase in the price of gin will tend to reduce the quantity of vermouth demanded (the price of vermouth remaining constant); in this case η_{xy} is negative and gin and vermouth are said to be complementary goods.

Linear demand surfaces for the two-good case are shown in Figures 3.4.1 and 3.4.2. In each graph, the quantity of X demanded is plotted on the vertical or "height" axis, while the prices of X and Y are plotted on the "width" and "length" axes. In each case, the plane $ABCD$ is the demand surface.

Figure 3.4.1 shows a linear demand surface for which goods X and Y are substitutes over the ranges of price considered. First, notice that the law of demand holds: as the price of X increases, its quantity demanded falls. Thus if the price of Y is held fixed at Op_{y1}, an increase in the price of X from Op_{x1} to Op_{x2} causes a decline in quantity demanded from RR' to TT'. Now hold the price of X constant at the amount Op_{x1}. As the price of Y increases from Op_{y1} to Op_{y2}, the quantity of X demanded rises from RR' to SS'—an increase in the price

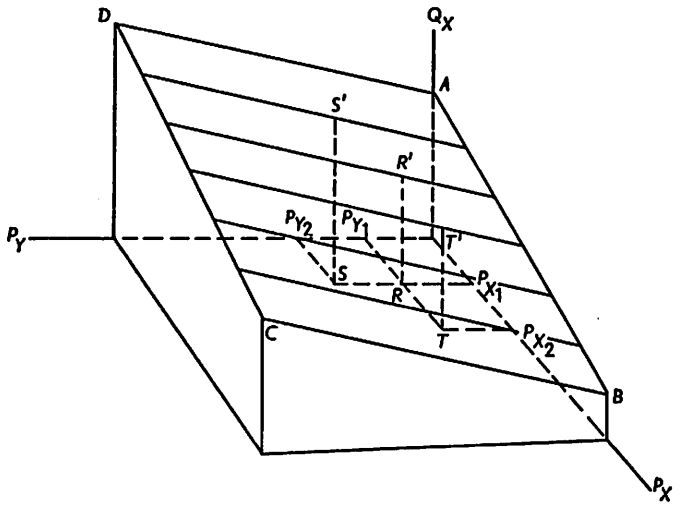

FIGURE 3.4.1

DEMAND SURFACE FOR GOOD X WHEN X AND Y ARE SUBSTITUTE GOODS

of good Y causes an increase in the quantity of X demanded. Thus the coefficient η_{xy} is positive, and the goods are said to be substitutes.

Figure 3.4.2 shows the opposite relationship over the range of prices considered. Again, first note that the law of demand obtains. For a fixed price of Y, Op_{y1}, an increase in the price of X from Op_{x1} to Op_{x2}

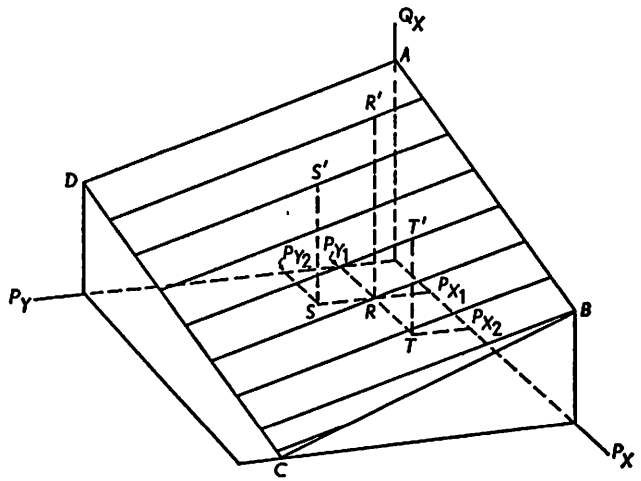

FIGURE 3.4.2

DEMAND SURFACE FOR GOOD X WHEN X AND Y ARE COMPLEMENTARY GOODS

causes a reduction in quantity demanded from RR' to TT'. But now hold the price of X constant at Op_{x1}. An increase in the price of Y from Op_{y1} to Op_{y2} also causes a decline in the quantity of X demanded, from RR' to SS' in this case. Accordingly, the coefficient of price cross-elasticity is negative and the commodities are said to be complementary goods.

The cross-elasticity approach to commodity classification directs attention to the change in quantity demanded resulting from a change in price *without* compensating for the change in the level of real income. The *total effect* of a price change is thus the criterion used in this classification scheme. On an empirical level, this is the only feasible method of commodity classification because market demand functions can be computed while individual preference functions cannot (from readily available data).

Furthermore, in applied problems, one is usually interested in the *market* relationship among commodities rather than the commodity relationship as viewed by an individual consumer. Thus the cross-elasticity classification of commodity relationships is *the one* most frequently encountered in applied studies. Indeed, reference to market cross-elasticities has even appeared in Supreme Court antitrust decisions.

3.4.b—Classification by Preference Functions

So far as one individual is concerned, however, commodities can be classified as substitute or complementary goods more accurately by reference to the preference function. For example, a very significant decrease in the price of beef might lead to an observed increase in the consumption of both beef and pork. The income effect of the price change, in this case, might more than offset the normally adverse substitution effect of an increase in the relative price of pork. The observed cross-elasticity would be negative, and one might be tempted to classify beef and pork as complementary goods. This temptation, however, should be resisted, for the result is attributable to an important change in real income.

As you have seen in Section 3.2, when there are only two goods the substitution effect is always negative. In other words, if there were only two goods they would necessarily be substitute goods. Complementarity can enter only through third markets. Let there be three commodities, good X, good Y, and what is often called "Hicks-Marshall" money (all other goods lumped together). Hicks defines substitute and complementary relationships in the following way.

Definition: Y is a substitute for X if the marginal rate of substitution of Y for money is diminished when X is substituted for money in such a way as to leave the consumer no better off than before.

Y is complementary with X if the marginal rate of substitution of Y for money is increased when X is substituted for money in such a way as to leave the consumer no better off than before.

The qualifying phrase in this definition restricts one to movements along the same *indifference surface*—since the definition requires three goods, it is not possible to illustrate the relationship by using two-

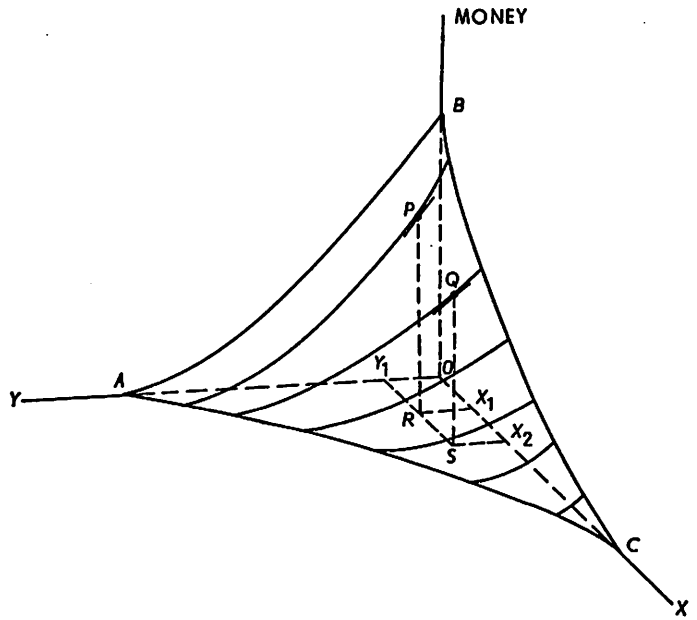

FIGURE 3.4.3

INDIFFERENCE SURFACE WHEN X AND Y ARE SUBSTITUTE GOODS

dimensional indifference *curves*. Indifference surfaces illustrating substitute and complementary goods are shown in Figures 3.4.3 and 3.4.4 respectively.

First consider Figure 3.4.3. One indifference surface is ABC. The individual is indifferent among all combinations of X, Y, and money represented by a point on the surface. For example, he is indifferent between OX_1 units of X, OY_1 units of Y, and RP units of money and the combination of OX_2 units of X, OY_1 units of Y, and SQ units of money. Let the consumer be situated at point P on the surface. The slope of the tangent drawn at that point is the marginal rate of substitution of Y for money. Now let the consumer substitute X for money so as to remain on the surface—he increases his consumption of

X from OX_1 to OX_2 and diminishes money from RP to SQ. The marginal rate of substitution of Y for money at point Q is given by the slope of the tangent drawn at that point. Since the slope at Q is less than the slope at P, commodities X and Y are substitute goods—when X is substituted for money so as to keep real income constant, the marginal rate of substitution of Y for money diminishes.

The meaning of Hicks' definition can be explained in two stages. The original equilibrium is at point P on the indifference surface ABC. Now let the price of X in terms of money fall (the price of money is

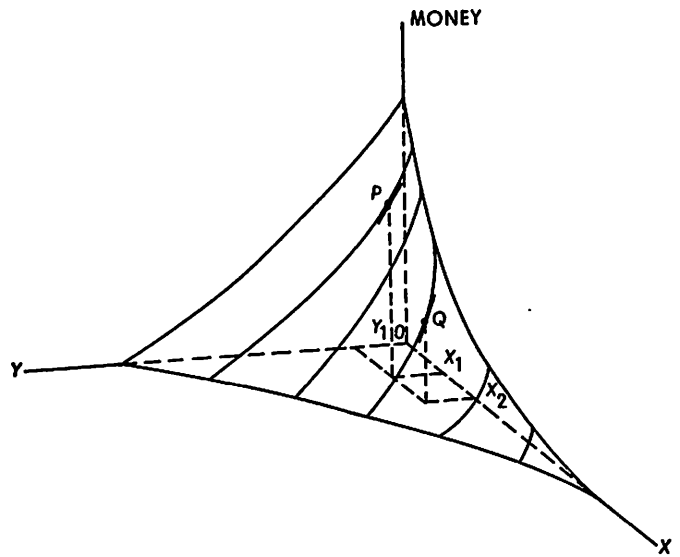

FIGURE 3.4.4

INDIFFERENCE SURFACE WHEN X AND Y ARE COMPLEMENTARY GOODS

unity), and let the consumer's real income be reduced so that he is restricted to the original indifference surface. The consumer substitutes X for money when its price falls, moving to a *temporary* equilibrium at point Q. But at point Q, the marginal rate of substitution of Y for money has decreased. The price of Y has not changed, so Q does not represent an ultimate equilibrium point because the marginal rate of substitution of Y for money does not equal the ratio of the price of Y to the price of money. To attain a final equilibrium, the consumer must curtail his consumption of Y (substitute X or money for Y) until the marginal rate of substitution of Y for money rises to the level of the unchanged price ratio.

An entirely analogous explanation applies to Figure 3.4.4. When

X is substituted for money, moving from point P to point Q, the marginal rate of substitution of Y for money increases. The two goods are complementary. Since the nominal price of Y has not changed (the Y-money price ratio is constant), and since the marginal rate of substitution of Y for money has increased, the consumer must increase his consumption of Y in order to attain a final equilibrium position. He must substitute Y for money until the marginal rate of substitution falls to the level of the unchanged price ratio.

Generalizing and simplifying somewhat, goods X and Y are substitutes if, after compensating for the change in real income incident to a change in the nominal price of X, a decrease in the price of X leads to a decrease in the quantity of Y consumed. Similarly, the two goods are complementary if an income-compensated decrease in the price of X leads to an increase in the quantity of Y consumed.

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Chapter 4 APPLICATIONS OF INDIFFERENCE CURVE ANALYSIS

4.1 INTRODUCTION

Whenever it can be supposed an individual has a preference ordering among two or more alternatives, it is possible to construct an indifference map depicting his preference function. The indifference map, in turn, can frequently be used to analyze various problems. In this chapter the indifference curve technique is used to analyze three theories closely related to the theory of demand: the theory of index numbers, the theory of exchange, and the determination of the individual supply of labor services. The latter two applications are repeated in Chapters 15 and 13 respectively.

4.2 THE THEORY OF EXCHANGE

The theory of exchange is a part of general equilibrium economics, a topic that will be developed in Part V. Given existing supplies of goods, the theory of exchange explains how goods are traded until an optimum allocation is attained.

The theory of exchange can be made as complex as one wishes by introducing large numbers of goods and of consumers. However, all essential elements of the theory can be obtained from a very simple model. To this end, suppose only two goods, X and Y, are available in fixed and instantaneously nonaugmentable supply. Further, suppose only two individuals, A and B, are in this simple model. Figure 4.2.1 shows the indifference-curve maps of both individuals for both commodities.

The indifference maps for A and B show the preferences of each; but that is as far as one can go with Figure 4.2.1. However, on the basis of the preference maps and the given supplies of goods, one can construct an "Edgeworth-Bowley" box diagram. The latter permits a complete analysis of the theory of exchange.

The Edgeworth-Bowley box diagram is constructed in Figure 4.2.2. The first step is to rotate the origin of individual B's indifference