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PART V

Theory of General Equilibrium and Economic Welfare

Well over 100 years ago Frederic Bastiat, a noted French economist, wrote about the Paris of his day. Hundreds of thousands of people then lived in Paris, each consuming a wide variety of commodities, especially food products not produced in the city. The survival of the city required the constant influx of goods and services. No single agency planned the daily inflow of commodities; but each day goods did arrive in approximately correct quantities: Paris survived. "Imagination is baffled when it tries to appreciate the vast multiplicity of commodities which must enter tomorrow in order to preserve the inhabitants from falling prey to the convulsions of famine, rebellion, and pillage," Bastiat wrote. "Yet all sleep, and their slumbers are not disturbed for a single minute by the prospect of such a frightful catastrophe."

Paris survived because of the unplanned cooperation of many people, most of whom competed against each other. Not for altruistic motives, to be sure, but for the profit to be gained from selling in the Paris market. Even before the days of Bastiat, Adam Smith had observed the effects of cooperation in production. Smith visited a small pin factory, one doubtlessly primitive by modern standards. Yet Smith was so struck by the gain in productivity resulting from cooperation and the specialization of labor that he wrote an account now classic in economic literature:

> One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to

whiten it is another; it is even a trade by itself to put them into paper. . . . I have seen a small factory of this kind where ten men only were employed and where some of them consequently performed two or three distinct operations. But though they were very poor and therefore but indifferently accommodated with the necessary machinery, they could, when they exerted themselves, make among them about twelve pounds of pins in a day. There are in a pound upwards of 4,000 pins of a middling size. Those ten persons, therefore, could

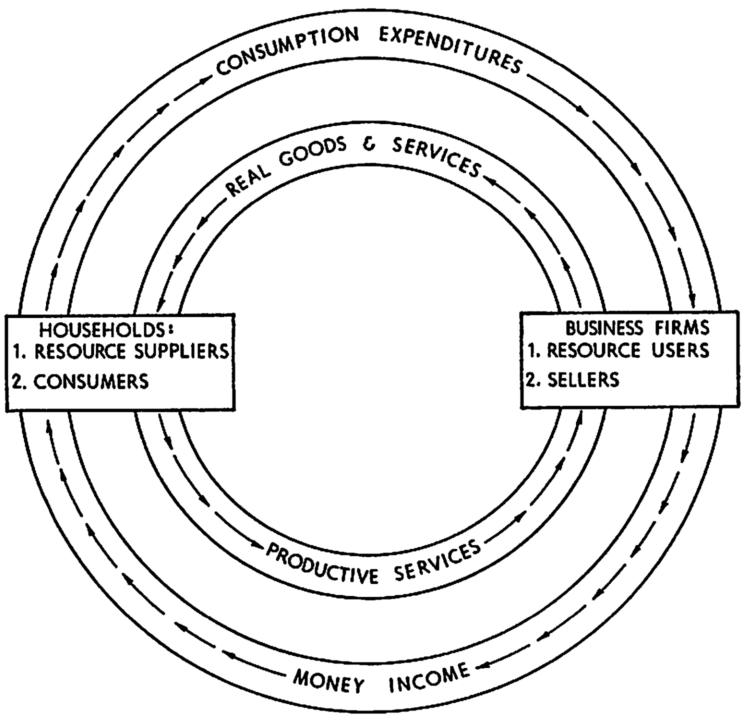


FIGURE V.1
CIRCULAR FLOW OF ECONOMIC ACTIVITY

make among them upwards of 48,000 pins a day. . . . But if they had all wrought separately and independently . . . they could certainly not each of them make twenty, perhaps not one pin in a day. . . .

Specialization and division of labor make possible a larger output than if each person worked alone and were self-sufficient. But self-sufficiency does guarantee that the consumer gets what he wants, or what he wants most and what is within his ability to achieve. When each person is not self-sufficient, the economy either must be *planned* by some central agency or there must be some mechanism that accomplishes the same goal. Adam Smith chose to call this mechanism the "invisible

hand;" in the terminology of today it might better be called a "great IBM machine in the sky." But whatever the terminology, a free-enterprise price system generally functions so as to achieve the goals of state planning, usually much more efficiently than planned economies achieve them. Economic welfare under a free-enterprise system is the topic of Chapter 16, after we begin a study of general economic equilibrium in Chapter 15.

To this point our discussion has focused only upon the economic behavior of single economic agents or of single industries or product groups. But there are millions of economic agents in the economy, and we have not yet seen how the behavior of each is coordinated to achieve a general equilibrium.

Looked at differently, the familiar graph in Figure V.1 illustrates the problem. On the one hand, households function both as consumers and resource suppliers. On the other hand, business firms use the resources, organize production, and sell the products of the process. There is a flow of *real* productive services from households to businesses and a return flow of *real* goods and services from business firms to households. If a barter system were feasible in an advanced industrial nation, we should have to go no further. But it is not; money must be introduced.

Rather than trade output for input, business firms pay households money income for the productive services supplied. In their role as consumer, households create a counterflow of consumption expenditures to business firms, exchanging their money income for the real goods and services supplied to them. Thus there is a monetary flow in one direction to offset each real flow in the opposite direction. The problem of general equilibrium analysis is to determine the process by which the various flows balance.

Chapter 15 THEORY OF GENERAL ECONOMIC EQUILIBRIUM

15.1 INTRODUCTION

According to the principle of maximization which has been adhered to throughout the book, each economic agent attains an equilibrium position when *something* is maximized. A consumer maximizes satisfaction subject to a budget constraint; an entrepreneur maximizes profit, possibly subject to the constraint imposed by a production function; workers may determine their labor supply curves by maximizing satisfaction derived from leisure, subject to given wage rates. In terms of an old cliche, we have studied the trees fairly intensively but we have not yet seen the forest.

The problem of forests arises, however. Millions of economic agents pursue their own goals and strive for their own equilibrium without particular regard for others. The problem is to determine whether the more-or-less independent behavior of all agents is consistent with each agent's attaining equilibrium. All economic agents, whether consumer, producer, or resource supplier, are *interdependent*; will *independent* action by each lead to a position in which equilibrium is achieved by all? This is the problem of general (static or stationary) economic equilibrium.

15.1.a—Quesnay's "Tableau Economique"

At this point a digression to history may be useful. Perhaps the earliest notion of stationary general equilibrium is in the work of a group of French economists called the "physiocrats." Foremost among them was an economist named Quesnay who, as early as 1758, presented a picture of general equilibrium by means of his "Tableau Economique."[1] Quesnay divided the economic agents of a society into three classes: the productive or agricultural class, the proprietary class, and the nonproductive class. He then suggested that the riches of a

[1] F. Quesnay, *Tableau economique et maximes generales du gouvernement economique* (Paris, 1758). For a later contribution, see J. Turgot, *Reflections sur la formation et la distribution des richesses* (Paris, 1776).

nation must be distributed among the three classes so as to attain a stationary (or flowing) equilibrium.

Quesnay's concept may be explained by an example taken from his book. Suppose there is a country with 130 million acres of land and 30 million people. The land properly tilled will produce 4 million units of food and 1 million units of raw material. Quesnay suggested that the "riches" must be distributed in the following manner. The productive class will retain 2 million units of food, which comprise the "avances annuelles" to sustain it during the next year. The productive class will pay 2 million units of food to the proprietary class as rent for the land, and trade the 1 million units of raw material to the nonproductive class (manufacturers) for 1 million units of manufactured goods. The proprietary class will retain 1 million units of food for subsistence and trade 1 million units to the nonproductive class for manufactured goods. Finally, the nonproductive class receives 1 million units of raw material from the productive class, transforms it into 3 million units of manufactured products, and exchanges 2 million units for 1 million units of raw materials from the productive class and 1 million units of food from the proprietary class.

Now we are back where we started; the system can continue to function in the same way year after year. It is a very simple model; perhaps it may seem trivial. Yet it represents a beginning point for general equilibrium theory. Or, to quote Fossati:

> It is against a background like this, which in essence is the idea of the stationary state, that the concept of the equilibrium of the economic system has been defined. . . . The stationary state is one in which every year the same processes are repeated, and the same distribution of goods takes place through the same channels. The stationary state of the "Tableau economique" is a model showing the conditions required for certain processes to function steadily and maintain each other in being on an unchanging scale indefinitely, like fountain-jets ever in movement yet always rising to the same height. It may not seem to offer a great contribution to the analysis of equilibrium, yet in view of the time of its formulation, it is a landmark in our science.[2]

15.1.b—Walras, Pareto, and Leontief

The theory of general equilibrium was developed much more thoroughly by the "Lausanne School," especially by Leon Walras and Vilfredo Pareto.[3] Concerned as it is with the individual equilibrium of

[2] Eraldo Fossati, *The Theory of General Static Equilibrium* (Oxford: Basil Blackwell, 1957), pp. 37–38.

[3] Leon Walras, *Elements d'Economie Politique Pure* (Lausanne: F. Rouge, 1874); for translation, see suggested readings at end of chapter. Vilfredo Pareto, *Cours d'Economie Politique* (Lausanne: F. Rouge, 1897).

millions of economic agents and the overall equilibrium of the system, the theory of general economic equilibrium has always been essentially mathematical in nature. Yet it has been given an operationally usable form by Leontief, resulting in his justly famous "input-output" analysis.[4]

15.1.c—Algebraic Statement of the Problem

In sections 15.2 and 15.3 a graphical treatment of general equilibrium is presented. However, to supplement this simple treatment of a simple economy, it seems desirable to set out an algebraic formulation of the general equilibrium model. It enables one to see that a solution of this complicated problem might exist; and it points forcefully to the general interdependence of all economic agents. Furthermore, the algebraic formulation shows that several important theoretical inferences may be drawn from the model, even though empirical implementation or verification of general or special (input-output) models of general equilibrium presents gigantic computation problems.

15.1.d—Equilibrium of Exchange

Let us begin with an extremely artificial case. Suppose there is a small, isolated country containing n individuals, each of whom possesses a well-defined parcel of land. These individuals truly resemble the lily of the valley, for they neither toil nor do they reap. They merely gather and exchange manna which, providentially enough, falls nightly upon their land. Indeed, m different types of manna fall in different concentrations on the various parcels of land. Every morning each individual gathers the various types of manna that have fallen upon his own ground.

For convenience, let us represent each individual by the subscript i. Thus with n individuals in the society, i takes the values $1, 2, \ldots, n$. In like manner, denote the different types of manna by the subscript j; this subscript accordingly has the values $1, 2, \ldots, m$. With this convention, we may let $\bar{x}_{ji}$ represent the quantity of type j manna gathered by individual i on a given morning; thus $\bar{x}_{1i}, \bar{x}_{2i}, \ldots, \bar{x}_{mi}$ is a complete list of individual i's initial manna holdings.

Our mythical society would be ideal except for the fact that the composition of the manna fall on each plot of land is not the composition desired by the individual who possesses the plot in question. That is, individual i wishes to have x_{ji} units of type j manna; but in fact he gathers $\bar{x}_{ji}$

[4] W. W. Leontief, *The Structure of the American Economy, 1919–1939* (New York: Oxford University Press, 1951).

units, and typically $\bar{x}_{ji} \neq x_{ji}$. Indeed, if $x_{ji} > \bar{x}_{ji}$, the individual must somehow obtain $x_{ji} - \bar{x}_{ji}$ units of type j manna from someone else if his consumption plan is to be realized. On the other hand, if $\bar{x}_{ji} > x_{ji}$, the individual may dispose of the surplus, $\bar{x}_{ji} - x_{ji}$, in the market. Of course, his consumption plan must be realistically constructed; he must plan to trade enough of some types of manna to finance his acquisition of other types.

For expository purposes it is convenient to play the exchange game in a way slightly different from that indicated above. In particular, suppose that each morning individual i first trades his entire stocks of manna of types $1, 2, \ldots, m-1$ in exchange for type m manna. After the initial exchanges, his entire manna holding is concentrated in type m manna; and the physical quantity of this manna stock represents his daily *real income.* Next, since it is difficult for us to deal with real income and for our imaginary citizens to engage in unrestricted barter, it is convenient to introduce "money" as a *unit of account* (but *not* as a medium of exchange or as a store of value).

To this end, let p_j represent the nominal market price of type j manna in terms of the unit of account. As it happens, the model of general equilibrium does not bring in the monetary side of the economy; as a consequence there is neither datum nor equation, such as the supply of money or the equation of exchange, to determine the *absolute* price level. Only *relative prices* are determined by the model—prices relative to the unit of account. Therefore, one must arbitrarily select some type of manna whose price is to be the unit of account or the numeraire for the system of relative prices. Since one may select the numeraire good and its unit price, let us specify type m manna as the numeraire and set its unit price at unity: $p_m = 1$. Using the notation just introduced, the monetary value of individual i's real income after the initial exchange is $\bar{x}_{mi}$; before the imaginary exchange it is $M = p_1\bar{x}_{1i} + p_2\bar{x}_{2i} + \ldots + \bar{x}_{mi}$ or, more compactly,

$$M = \sum_{j=1}^{m} p_j \bar{x}_{ji} .$$

Next, suppose individual i allocates the monetary value of his real income by trading for the various types of manna he desires. Since x_{ji} represents his barter purchases of type j manna, allocating his entire money income means that $M = p_1x_{1i} + p_2x_{2i} + \ldots + x_{mi}$ or, more simply,

$$M = \sum_{j=1}^{m} p_j x_{ji} .$$

Since the two M's are equal, or the monetary value of his income is equal to the monetary value of the things he wishes and is able to command, the *budget constraint* for individual i may be written as

$$(15.1.1) \qquad \sum_{j=1}^{m} p_j(x_{ji} - \bar{x}_{ji}) = 0 .$$

As in Part I of the text, we assume that the level of satisfaction attained by any consumer depends upon the quantities of the various types of manna he consumes. In particular, suppose the level of satisfaction attained by individual i may be represented by the ordinal preference function

$$(15.1.2) \qquad u_i = u_i(x_{1i}, x_{2i}, \ldots, x_{mi}) .$$

Still following the development of the theory of consumer behavior in Part I, assume that individual i ($i = 1, 2, \ldots, n$) attempts to maximize his satisfaction (expression 15.1.2) subject to his budget constraint (expression 15.1.1). As you already know, maximization requires a condition that may be expressed in either of two ways: (a) the marginal rate of substitution in consumption between any two types of manna must be equal to the ratio of their prices; or (b) the marginal utility of a dollar's worth of one type of manna must equal the marginal utility of a dollar's worth of every other type.

Let u_{ji} denote the marginal utility of type j manna to individual i. The consumer-maximization requirement stated in (b) above may be expressed as

$$\frac{u_{1i}}{p_1} = \frac{u_{2i}}{p_2} = \cdots = u_{mi} ,$$

since $p_m = 1$. Furthermore, this same relationship must hold for each individual in the economy. Utilizing this knowledge, we may write out a set of equations that formally describes the *exchange equilibrium* in a system of general equilibrium:

$$(15.1.3) \qquad \begin{aligned} &\frac{u_{ji}}{p_j} = u_{mi} , \quad \begin{matrix} (j = 1, 2, \ldots, m-1) \\ (i = 1, 2, \ldots, n) \end{matrix} \\ &\sum_{j=1}^{m} p_j(x_{ji} - \bar{x}_{ji}) = 0 . \qquad (i = 1, 2, \ldots, n) . \end{aligned}$$

To reiterate: equations (15.1.3) imply that each individual in the economy exchanges manna with other individuals until all individuals as consumers are in a satisfaction-maximizing equilibrium. The first subset of equations states that for general equilibrium to obtain, *each individual* ($i = 1, 2, \ldots, n$) must finally hold manna in such proportions that the marginal utility of a dollar's worth of one type of manna is the same as the marginal utility of a dollar's worth of every other type of manna. The second subset of equations stipulates that each individual must abide by his budget constraint.

15.1.e—Equilibrium of Production

As fanciful as the manna-exchange model might seem to be, it actually applies to an ever-expanding segment of the American economy. More specifically, the problem confronting welfare recipients is precisely the same as that confronting manna gatherers: a consumer in either group must allocate his *given* money income so as to maximize satisfaction. Nonetheless, to present a formal model of the entire economy the exchange model must be supplemented by a model of production. Let us therefore turn to the production side of the model of general economic equilibrium.

Let there be q business firms in the economy, and denote them by the subscript s; thus s takes the values $1, 2, \ldots, q$. Our x symbols must now represent both commodities and resources. Firm s uses various resources and commodities to produce one or more commodities. As in Part II, we assume that there is a production function showing the output possibilities for each commodity. For example, firm s might produce commodity two by using resources five, six, and seven. We may thus write a production function for firm s, such as

$$x_{2s} = f_s(x_{5s}, x_{6s}, x_{7s}) .$$

However, to allow for the production of several commodities by means of commodities and resources, it is more convenient to write a *transformation function* for each firm. The transformation function is based upon the underlying production functions for each commodity; it merely shows that one set of commodities and resources can be transformed into another set by the production process. The transformation function for firm s is written as

$$(15.1.4) \qquad f_s(x_{1s}, x_{2s}, \ldots, x_{ms}) = 0 .$$

In equation (15.1.4) the x symbols represent both outputs and inputs. Thus we must adopt another convention: (a) if x_{js} represents the output of commodity j by firm s, it is a *positive* quantity; and (b) if x_{ks} represents the input of commodity or resource k into the production process of firm s, it is a *negative* quantity.

Let us now return briefly to the example of the firm that produces commodity two by using inputs five, six, and seven. The total revenue of the firm (say, firm s) is p_2x_{2s}; its total cost is $p_5x_{5s} + p_6x_{6s} + p_7x_{7s}$. Accordingly, the profit of firm s, denoted r_s, is $r_s = p_2x_{2s} + p_5x_{5s} + p_6x_{6s} + p_7x_{7s}$, since x_{5s}, x_{6s}, and x_{7s} are all negative. That is, since each of these is itself negative, a *plus* sign must be used in the profit equation in order to subtract cost from revenue. Generalizing to the transformation function in equation (15.1.4.), the profit accruing to firm s may be expressed as

$$r_s = \sum_{j=1}^{m} p_j x_{js} \,. \tag{15.1.5}$$

As in Parts II and III, we assume that each firm attempts to maximize profit (expression 15.1.5) subject to the constraint imposed by its transformation function (15.1.4). The maximization process leads to the following set of equations, which formally describes the production side of the economy:

$$(15.1.6) \quad \begin{aligned} &\frac{f_{js}}{p_j} = f_{ms}\,, \qquad (j = 1, 2, \ldots, m-1) \\ &\qquad\qquad\qquad (s = 1, 2, \ldots, q) \\ &f_s(x_{1s}, x_{2s}, \ldots, x_{ms}) = 0\,. \qquad (s = 1, 2, \ldots, q) \end{aligned}$$

You already know the meaning of the equations in the first subset of (15.1.6), even though it may take quite a bit of explanation before the conditions become familiar. If both j and m represent inputs, f_{js} and f_{ms} are the total marginal products of input j and input m respectively (for example, f_{js} shows the increase in all outputs of firm s attributable to the addition of one unit of input j in the production process). In this case, the first subset of equations states the conditions for economically efficient operation. In particular, the marginal product of a dollar's worth of any one input must equal the marginal product of a dollar's worth of every other input. Expressed alternatively, the marginal rate of technical substitution between any two inputs must be equal to the ratio of their prices.

To explain the meaning of the first subset of equations when j and m are outputs, and when j is an output and m an input, requires somewhat more discussion. To begin, suppose both j and m are outputs of firm s. A typical equation of the first subset may then be written $f_{js}/f_{ms} = p_j/p_m = p_j$, since p_m equals one by hypothesis. The expression f_{js}/f_{ms} is called the marginal rate of transformation of output j into output m. It shows the amount by which the output of commodity m may be expanded if the output of commodity j is reduced by one unit, given the volume of inputs available and used.

A little reflection will convince the reader that the marginal rate

of transformation equals the ratio of the marginal cost of j to the marginal cost of m. First, the value of the inputs saved when the output of j is reduced by one unit is, by definition, the marginal cost of commodity j. Next, the same inputs are used to produce additional units of m. If the output of m increases by two units (the marginal rate of transformation is 2:1), then on average the marginal cost of producing m at the current rate of output must be one half the marginal cost of producing j. If we let the marginal cost of commodity j be, say, ten, the marginal cost of m must be five. Therefore, f_{js} is proportional to ten, f_{ms} is proportional to five, and the ratio of marginal costs is two, which is precisely the same as the marginal rate of transformation.

In this light, the maximization conditions simply state that if the marginal cost of producing commodity j is twice as great as the marginal cost of producing commodity m, the price of commodity j must also be twice as great as the price of commodity m. More generally, when both j and m represent outputs, the first subset of equations imposes requirements that may be stated in three alternative ways: (a) the marginal rate of transformation must equal the price ratio; (b) the ratio of marginal costs must equal the price ratio; and (c) the ratio of marginal cost to price must be the same for each commodity produced. The last alternative ties in closely with perfect competition, for under that form of market organization price equals marginal cost for each commodity.

The reason behind this requirement may easily be explained. Suppose price equals marginal cost in the production of commodity m, but the price of j exceeds its marginal cost. Clearly, the entrepreneur can increase his profit by shifting resources from the production of m into the production of j. Since the output of m declines, its marginal cost declines also; and the marginal cost of j rises as its output expands. The entrepreneur can increase his profit by shifting resources from m to j until the point is reached at which the marginal cost-price ratio is the same for both commodities.

Finally, consider the case in which j is an output and m is an input. The maximizing conditions in the first subset of equations may then be written

$$p_m = p_j \frac{f_{ms}}{f_{js}} \,.$$

As you will recall, f_{ms} is the addition to the outputs of all commodities produced by firm s attributable to the addition of one unit of input m into the production process. The ratio f_{ms}/f_{js} is the marginal product of input m in the production of output j alone. The condition thus states that the price of the input must be equal to the value of its marginal product.

Now return to equations (15.1.6). The first subset of equations has been explained from the standpoint of one business firm. The entire subset requires that the described conditions hold for all firms and for all pairs of outputs, for all pairs of inputs, and for all combinations of an output and an input. The second subset stipulates that each firm must operate subject to the limitations imposed by the technical conditions of production.

15.1.f—Perfect Competition

The model of general equilibrium may be expanded so as to include some aspects of imperfect competition. However, the present treatment is restricted to the perfectly competitive model as originally developed by Walras. If we further restrict our analysis to long-run equilibrium under perfect competition, entrance into and exist from industries by business firms reduces pure profit to zero. Hence r_s in equation (15.1.5) is zero, and the budget constraints in equations (15.1.1) do not have to be modified to allow for the existence of profit.

15.1.g—Market Equilibrium

Equations (15.1.3) and (15.1.6) impose certain equilibrium conditions upon each individual and each business firm operating in the economy. However, to insure that a *general* equilibrium is in fact attained, a set of market conditions must also be imposed. More specifically, we must require that quantity demanded equals quantity supplied for each resource and each commodity.

The net amount of commodity j demanded (if positive) or supplied (if negative) by individuals is

$$\sum_{i=1}^{n}(x_{ji} - \bar{x}_{ji}) .$$

Similarly, the net amount of commodity j supplied (if positive) or demanded (if negative) by business firms is

$$\sum_{s=1}^{q} x_{js} .$$

Thus the market condition that quantity demanded equals quantity supplied for all items traded is represented by[5]

[5] In equation (15.1.7), j varies over all commodities and there are actually m equations. However, it can be shown that one equation is not independent. Hence j is shown varying from one to $m-1$.

$$\sum_{s=1}^{q} x_{js} = \sum_{i=1}^{n}(x_{ji} - \bar{x}_{ji}) . \qquad (j = 1, 2, \ldots, m-1). \tag{15.1.7}$$

15.1.h—General Economic Equilibrium

Now let us take stock. Equations (15.1.3) show the conditions that must hold if each consumer is to maximize satisfaction subject to his budget constraint. Similarly, equations (15.1.6) show the profit-maximizing conditions for each firm. Finally, equations (15.1.7) give the market equilibrium conditions on quantities demanded and supplied. We now have all the trees; do they constitute a forest?

The algebraic formulation of the general equilibrium model is obtained by combining these three sets of equations, repeated below:

$$\text{(i)} \qquad \frac{u_{ji}}{p_j} = u_{mi} , \qquad \begin{matrix}(j = 1, 2, \ldots, m-1)\\(i = 1, 2, \ldots, n)\end{matrix}$$

$$\text{(ii)} \qquad \sum_{j=1}^{m} p_j(x_{ji} - \bar{x}_{ji}) = 0 , \qquad (i = 1, 2, \ldots, n)$$

$$\text{(iii)} \qquad \frac{f_{js}}{p_j} = f_{ms} , \qquad \begin{matrix}(j = 1, 2, \ldots, m-1)\\(s = 1, 2, \ldots, q)\end{matrix}$$

$$\text{(iv)} \qquad f_s(x_{1s}, x_{2s}, \ldots, x_{ms}) = 0 , \qquad (s = 1, 2, \ldots, q)$$

$$\text{(v)} \qquad \sum_{s=1}^{q} x_{js} = \sum_{i=1}^{n}(x_{ji} - \bar{x}_{ji}) . \qquad (j = 1, 2, \ldots, m-1)$$

If a general equilibrium exists, the equations above may be solved for all of the economic variables. In particular, equations (i) and (ii) give *individual demands* x_{ji} in terms of the prices; thus they give aggregate demand for each commodity as well. Furthermore, equations (iii) and (iv) give *production by firms* x_{js} in terms of the prices, so the aggregate supply of each commodity is also given. Finally, equations (v) guarantee that quantity demanded equals quantity supplied in each market.

But none of this tells us whether a general economic equilibrium exists. At this level of analysis, the best we can do is to count variables and equations and thereby determine whether the model is consistent with the existence of a general economic equilibrium.

The variables in the system (i)–(v) are: (a) the mn quantities demanded x_{ji} of the m goods by the n individuals; (b) the mq quantities supplied x_{js} of the m commodities by the q firms; and (c) the $m-1$ prices p_j (remembering that p_m equals one by definition). The general equilibrium model, therefore, contains $mn + mq + (m - 1)$ variables. Now count equations: set (i) provides $n(m - 1)$; set (ii) contains n; set (iii) has $q(m - 1)$; set (iv) provides q; and set (v) gives an additional $m - 1$. Thus there are $n(m - 1) + n + q(m - 1) + q + (m - 1)$ equations, or more simply, $mn + mq + (m - 1)$. The number of unknowns precisely equals the number of equations; thus a consistent and determinate solution may exist.

In other words, a general economic equilibrium may exist; the independent behavior of interdependent economic agents may provide a consistent equilibrium. On a formal level, quite a number of behavioral relationships were deduced and the process of economic interaction illustrated. To be sure, the discussion was restricted to a highly simplified economy, more especially because monetary problems were removed by the assumption that "money" exists only as a unit of account and not a medium of exchange. Yet even in this simplified setting the theory of general equilibrium is complex; small wonder we concentrate primarily on the partial economic equilibrium of individual economic agents rather than on the general economic equilibrium of interdependent agents.

15.2 GENERAL EQUILIBRIUM OF EXCHANGE

The algebraic formulation of subsections 15.1.c–15.1.h may cast general equilibrium theory in somewhat the wrong light. Thus the basic general equilibrium model is now given a graphical treatment. In the simplest models a good bit of insight into the economic process can be obtained. To be sure, the model used is simple, perhaps naively so; nonetheless it depicts the fundamental essentials of a multiperson economy in which individual behavior is not coordinated by a central planning agency.

At the outset we consider an economy in which exchange but not production takes place. There are only two people and only two goods. Each person has an *initial endowment* of each good, but he does not necessarily have the goods in the proportion that yields him greatest satisfaction. If not, some exchange of commodities between individuals will arise.[6] To analyze exchange, and other problems as well, we need to develop a graphical device known as the Edgeworth box diagram.

15.2.a—Edgeworth Box Diagram

The Edgeworth box diagram is a graphical technique for illustrating the interaction between two economic activities when their inputs

[6] See Chapter 4.2 for a previous treatment.

are fixed in quantity. It is thus an ideal instrument for analyzing general equilibrium and economic welfare.

Two basic Edgeworth box diagrams are illustrated in Figure 15.2.2. Panel a shows the construction for a consumption problem whose inputs are types of, say, food; panel b refers to production activities whose inputs are factors of production.

First consider Figure 15.2.1. There are two consumption goods, X and Y; these goods are available in absolutely fixed amounts. In

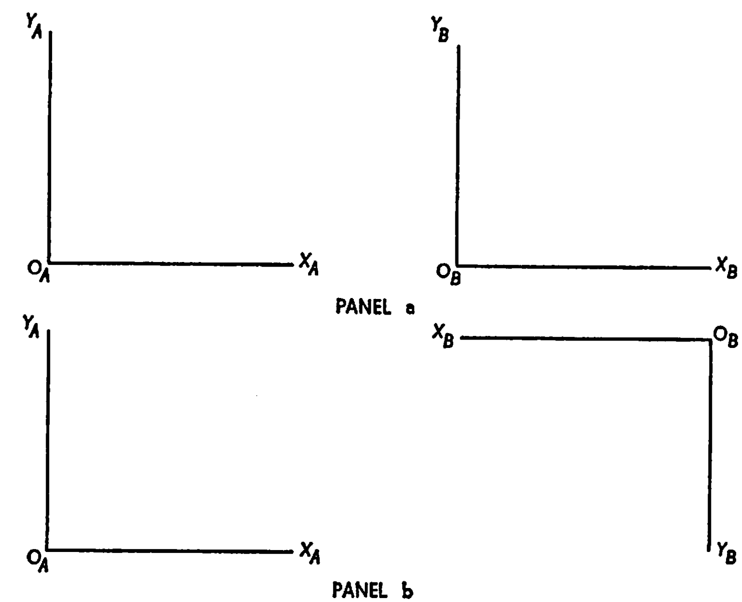
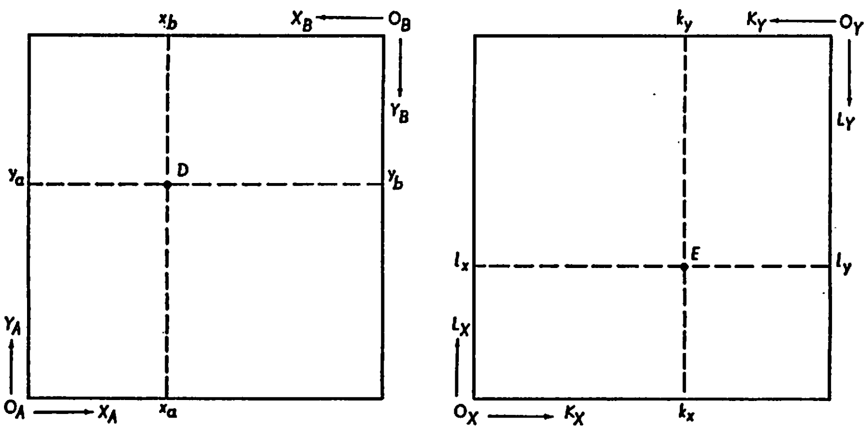


FIGURE 15.2.1

CONSTRUCTING THE EDGEWORTH BOX DIAGRAM FOR A CONSUMPTION PROBLEM

addition, only two individuals are in the society, A and B; they initially possess an endowment of X and Y, but the endowment ratio is not the one either would choose if he were allowed to specify it. This general equilibrium problem is graphically illustrated by constructing an *origin* for A, labeled O_A, and plotting quantities of the two goods along the abscissa and ordinate. Thus from the origin O_A, the quantity of X held by $A(X_A)$ is plotted on the abscissa and the quantity of $Y(Y_A)$ on the ordinate. A similar graph for B, with origin O_B, may be constructed beside the graph for A. These two basic graphs are illustrated in panel a, Figure 15.2.1.

Next, rotate the B-graph 180 degrees to the left, so that it is actually "upside down" when viewed normally, as shown in panel b. The Edgeworth box diagram is formed by bringing the two graphs together. There could conceivably be a problem involving the lengths of the axes; if the X axes meshed, the Y axes might not. The problem does not in fact exist, however, because of our assumption concerning fixed availabilities of X and Y. X_A plus X_B must equal X, and Y_A plus Y_B must equal Y. The length of each axis measures the fixed quantity of the good it represents; when the two "halves" in panel b are brought together both axes mesh. One thus obtains panel a, Figure 15.2.2.



PANEL a—EDGEWORTH BOX DIAGRAM FOR CONSUMPTION PROBLEM

PANEL b—EDGEWORTH BOX DIAGRAM FOR PRODUCTION PROBLEM

FIGURE 15.2.2

EDGEWORTH BOX DIAGRAM

The point D in panel a indicates the initial endowment of X and Y possessed by A and B. A begins with O_Ax_A units of X and O_Ay_A units of Y. Since the aggregates are fixed, B must originally hold $O_Bx_B = X - O_Ax_A$ units of X and $O_By_B = Y - O_Ay_A$ units of Y.

In a similar fashion, not illustrated in detail, one may construct an Edgeworth diagram for a production problem. The finished product is shown in panel b, Figure 15.2.2. Two goods, X and Y, are produced by means of two inputs, K and L. The two inputs are fixed in aggregate quantity. The origin of coordinates for good X is O_X, for good Y is O_Y. The quantities of inputs of K and L in producing X and Y are plotted along the axes. Accordingly, any point in the box represents a particular allocation of the two inputs between the two production processes. At point E, for example, O_Xk_X units of K and O_Xl_X units of L are used in

producing X. As a consequence, $O_Yk_Y = K - O_Xk_X$ units of K, and $O_Yl_Y = L - O_Xl_X$ units of L, are allocated to the production of Y.

15.2.b—Equilibrium of Exchange

As a first step toward general equilibrium analysis, consider an economy in which exchange of initial endowments takes place. For the moment, production is ignored. If you like, you may think of the problem in the following context. A small country exists with two inhabitants, A and B, each of whom owns one half the land area. As in our other manna example, A and B neither sow nor reap; they merely gather manna of types X and Y that fall nightly. Each gathers the manna that falls on his land; but the two types do not fall uniformly. There is a

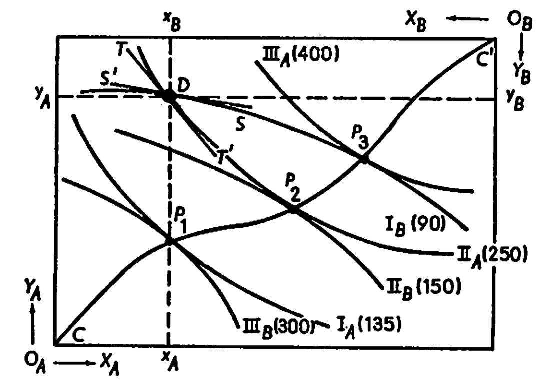


FIGURE 15.2.3

GENERAL EQUILIBRIUM OF EXCHANGE

relatively heavy concentration of Y-manna on A's property and, consequently, a relatively heavy concentration of X-manna on B's land.

The problem of exchange is analyzed by means of the Edgeworth box diagram in Figure 15.2.3. To the basic box diagram, whose dimensions represent the nightly precipitation of manna, we add indifference curves for A and B. For example, the curve I_A shows combinations of X and Y that yield A the same level of satisfaction. In ordinary fashion, II_A represents a greater level of satisfaction than I_A; III_A than II_A; and so on. Quite generally, A's well being is enhanced by moving toward the B origin; B, in turn, enjoys greater satisfaction the closer he moves toward the A origin.

Suppose the initial endowment (the nightly fall of manna) is point D; A has O_Ax_A units of X and O_Ay_A units of Y. Similarly, B has O_Bx_B and O_By_B units of X and Y respectively. The initial endowment places A

on his indifference curve II_A and B on his curve I_B. At point D, A's marginal rate of substitution of X for Y, given by the slope of TT', is relatively high; A would be willing to sacrifice, say, three units of Y in order to obtain one additional unit of X. At the same point, B has a relatively low marginal rate of substitution, as shown by the slope of SS'. Or turning it around, B has a relatively high marginal rate of substitution of Y for X. He may, for example, be willing to forego four units of X to obtain one unit of Y.

A situation such as this will always lead to exchange if the parties concerned are free to trade. From the point D, A will trade some Y to B, receiving X in exchange. The exact bargain reached by the two traders cannot be determined. If B is the more skillful negotiator, he may induce A to move along II_A to the point P_2. All the benefit of trade goes to B, who jumps from I_B to II_B. Just oppositely, A might steer the bargain to point P_3, thereby increasing his level of satisfaction from II_A to III_A, B's real income remaining I_B. Starting from point D, the ultimate exchange is very likely to lead to some point between P_2 and P_3; but the skill of the bargainers and their initial endowments determine the exact location.

One important thing can be said, however. Exchange will take place until the marginal rate of substitution of Y for X is the same for both traders. If the two marginal rates are different, one or both parties can benefit from exchange; neither party need lose. In other words, the exchange equilibrium can occur only at points such as P_1, P_2, and P_3 in Figure 15.2.3. The locus CC', called the contract or conflict curve, is a curve joining all points of tangency between one of A's indifference curves and one of B's. It is thus the locus along which the marginal rates of substitution are equal for both traders. We accordingly have the following:

Proposition: the general equilibrium of exchange occurs at a point where the marginal rate of substitution between every pair of goods is the same for all parties consuming both goods. The exchange equilibrium is not unique; it may occur at any point along the contract curve (for multiple traders, it is more properly called the contract hypersurface).

The contract curve is an optimal locus in the sense that if the trading parties are located at some point not on the curve, one or both can benefit, and neither suffer a loss, by exchanging goods so as to move to a point on the curve. To be sure, some points not on the curve are preferable to some points on the curve. But for any point not on the curve, one or more attainable points on the curve are preferable.

15.2.c—Deriving the Utility-Possibility Frontier

The contract curve is an optimal locus in *commodity space*; it shows all pairs of allocations of X and Y to A and B such that the marginal rate of substitution is equal for both parties. This exchange equilibrium locus can be transformed from commodity space to utility space, obtaining what is called the *utility-possibility frontier* relative to the particular endowment aggregate in Figure 15.2.3. The process of derivation is illustrated in Figure 15.2.4.

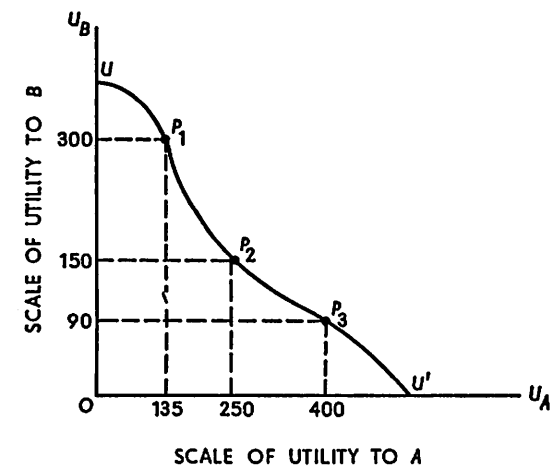


FIGURE 15.2.4

DERIVING THE UTILITY-POSSIBILITY FRONTIER FROM THE CONTRACT CURVE

First consider the point P_1 in Figure 15.2.3. In A's scale of utility measurement, all points on I_A are valued at 135; thus P_1 is associated with a utility value of 135. Similarly, in B's utility scale, all points along III_B have the value 300. Now construct a graph, as in Figure 15.2.4, whose coordinate axes are A's and B's utility scales. The point P_1, with coordinates 135 and 300, can be plotted on this graph. Similarly, all other points along the contract curve in commodity space can be plotted in utility space by noting the pair of utility values associated with each point of tangency. Connect all such points by a curve, labeled UU' in Figure 15.2.4. This curve is the utility-possibility frontier.

Definition: the utility-possibility frontier is the locus showing the maximum level of satisfaction attainable by one trading party for every given level of satisfaction of the other. The curve so generated depends upon the absolute endowment of each commodity and upon the aggregate commodity endowment ratio—that is, upon X, Y, and Y/X.[7]

15.3 GENERAL EQUILIBRIUM OF PRODUCTION AND EXCHANGE

The topic of general equilibrium has been introduced by discussing a model in which production does not occur; consumers simply exchange existing stocks or endowments of commodities. We shall now expand by adding a production side to the basic model. There are still only two consuming units in the society, A and B; there are also only two *producible* commodities, X and Y. But now they must be produced by means of two inputs, K and L. The production functions for X and Y are assumed given, and there are fixed, nonaugmentable quantities of the inputs K and L. In other words, the initial endowments in the present model are the fixed input supplies rather than fixed quantities of the two consumption goods.

15.3.a—General Equilibrium of Production

The analysis of the general equilibrium of production is precisely the same as that of the general equilibrium of exchange. The only difference is terminology (economic jargon). The fixed endowments of inputs K and L determine the dimensions of the Edgeworth box diagram in Figure 15.3.1. Next, the given and unchanging production functions for goods X and Y enable us to construct the isoquant maps for each, illustrated by such curves as II_X and III_Y.

Suppose inputs are originally allocated between production of X and Y such that $O_X k_X$ units of K and $O_X l_X$ units of L are used in making X; the remainder, $O_Y k_Y$ and $O_Y l_Y$ units of K and L respectively, is used in producing Y. This allocation is represented by point D in the Edgeworth box—the point at which II_X intersects II_Y. At the allocation D, the marginal rate of technical substitution of K for L in producing X, given by the slope of SS', is relatively high. The marginal product of K in producing X is high relative to the marginal product of L. The II_X level of production can be maintained by substituting a relatively small amount of K for a relatively larger amount of L. The opposite situation prevails in Y production, as shown by the slope of TT'. The marginal rate of technical substitution of K for L in producing Y is relatively low; thus a comparatively large amount of K can be released by substituting

[7] The student should remember that the utility numbers are purely arbitrary so far as interpersonal utility comparisons are concerned. In particular, 300 for B is not necessarily greater than 135 for A, although to A, 136 is greater than 135.

a relatively small amount of L while maintaining the II_Y level of output.

If the producer of X at point D substitutes one unit of K he can, let us suppose, release two units of L. The producer of Y, by employing the two units of L released from X production, can maintain output and release, let us suppose, four units of K. Thus from a point such as D input substitution by producers will enable the society to move to P_2, P_3, or any point in between. At P_2, the output of X is the same as at D but the output of Y has been increased to the III_Y level. If the movement is to P_3, the output of X increases with no change in the volume of Y production.

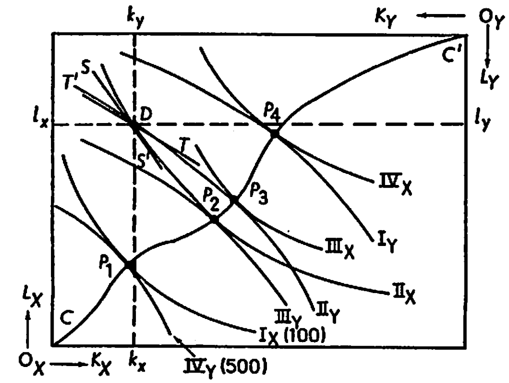


FIGURE 15.3.1

GENERAL EQUILIBRIUM OF PRODUCTION

The foregoing discussion establishes a pervasive principle. Whenever the marginal rate of technical substitution between two inputs is different for two producers, one or both outputs may be increased, and neither decreased, by making the appropriate input substitutions. In the example in Figure 15.3.1, the X producer would substitute K for L, decreasing the marginal product of K, increasing that of L, and thereby lowering the marginal rate of technical substitution. The producer of Y, on the other hand, should substitute L for K, with the opposite results. Production of one or both goods can always be increased without an aggregate increase in inputs unless the marginal rates of technical substitution between the inputs are the same for both producers.

The locus CC', again called the *contract* or *conflict curve*, is a curve showing all input allocations that equalize the marginal rates of technical substitution—that is, the locus of tangencies between an X isoquant and a Y isoquant. We can accordingly state the following:

Proposition: the general equilibrium of production occurs at a point where the marginal rate of technical substitution between every pair of inputs is the same for all producers who use both inputs. The production equilibrium is not unique; it may occur at any point along the contract curve.

The contract curve is an optimal locus in the sense that if the producers are located at a point not on the curve, the output of one or both commodities can be increased, and the output of neither decreased, by making input substitutions so as to move to a point on the curve. To be sure, some points not on the curve correspond to a greater aggregate output than some points on the curve. But for any point not on the curve there are one or more attainable points on the curve associated with a greater aggregate output.

15.3.b—General Equilibrium of Production and Exchange

For any input endowment there are an infinite number of potential production equilibria, any point on the contract curve in Figure 15.3.1. Each point represents a particular volume of output of X and of Y, and thereby dictates the dimensions of an Edgeworth box diagram for exchange (such as Figure 15.2.3). Furthermore, each consumption-exchange box leads to an infinite number of potential exchange equilibria, any point on the contract curve associated with the box in question. Accordingly, there are a multiple infinity of potential general equilibria of production and exchange.

The objective of any society is to attain that particular general equilibrium which maximizes the economic welfare of its inhabitants. As we shall see in Chapter 16, there are ways by which either a free-enterprise system or a decentralized socialist state may attain the optimum.

15.3.c—Deriving the Production-Possibility Frontier or Transformation Curve

The contract curve associated with the general equilibrium of production is a locus of points in *input space*; the curve shows the optimal output of each commodity corresponding to every possible allocation of K and L between X and Y. With the allocation of inputs indicated by point P_1 in Figure 15.3.1, 500 units of Y and 100 units of X are the maximum attainable production. By constructing a graph whose coordinate axes show the quantities of X and Y produced, and plotting the output pairs corresponding to each isoquant tangency in Figure 15.3.1, one may generate the curve labeled TT' in Figure 15.3.2.

The curve so obtained is called the production-possibility frontier or the transformation curve.

The transformation curve is obtained by mapping the contract curve from input space into output space. Fundamentally, this locus depicts the choices a society can make. It shows, in other words, the various (maximum) combinations of X and Y that are attainable from the given resource base (input endowment). No output combination

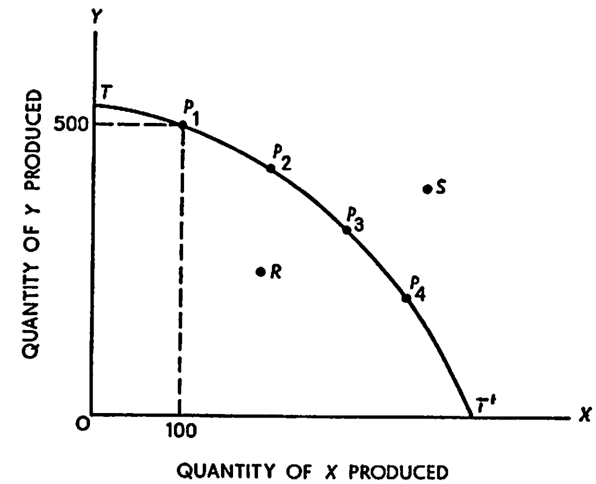


FIGURE 15.3.2

DERIVING PRODUCTION-POSSIBILITY FRONTIER FROM THE CONTRACT CURVE

represented by a point lying outside the production-possibility frontier (such as S) can be attained; such a level of output would require a greater resource base. On the other hand, a point lying inside the locus (such as R) is neither necessary nor desirable; it would entail a needless sacrifice of goods attributable to unemployment of available resources. Thus one objective of a society is to attain an equilibrium position *on*, not below, its production-possibility frontier.

Definition: the production-possibility frontier or transformation curve is a locus showing the maximum attainable output of one commodity for every possible volume of output of the other commodity, given the fixed resource base. The curve so generated depends upon the absolute endowment of each resource, upon the aggregate input endowment ratio, and upon the "state of the art" (the production functions for both goods).[8]

[8] For a numerical example of the derivation of the transformation curve, and a simple mathematical formulation, see C. E. Ferguson, "Transformation Curve in Production Theory: A Pedagogical Note," *Southern Economic Journal*, XXIX (1962), pp. 96–102.